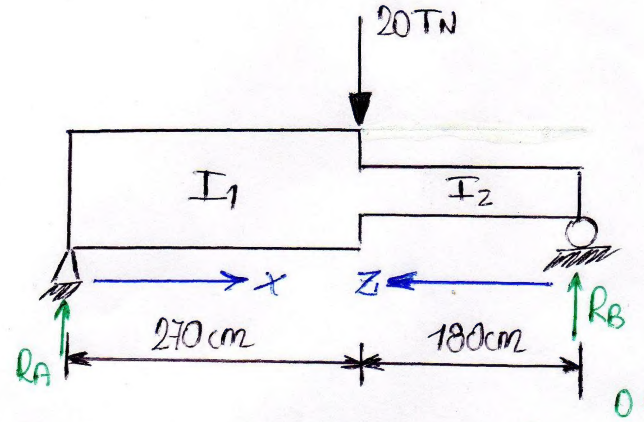
Doble integración

2.- Dada la viga simplemente apoyada de momento de inercia variable, determine el giro y la flecha en el punto de la carga aplicada por el momento de doble integración. Se conoce $I_1 = 16650\ cm^4$ $I_2 = 12500\ cm^4$. $E = 2\times10^6\ Kf/cm^2$



$20(270) = 450 R_B$

$12TN = R_B$

$8TN = R_A$

$0 \leq x \leq 270$

$EI_1 y_1'' = R_A x$

$EI_1 y_1' = \frac{R_A x^2}{2} + C_1$

$EI_1 y_1 = \frac{R_A x^3}{6} + C_1 x + C_2$

Para $x=0$ $y_1 = 0$ $C_2 = 0$

$0 \leq z \leq 180$

$EI_2 y_2'' = R_B z$

$EI_2 y_2' = \frac{R_B z^2}{2} + C_3$

$EI_2 y_2 = \frac{R_B z^3}{6} + C_3 z + C_4$

Para $z=0$ $y_2 = 0$ $C_4 = 0$

1ERA CONDICION FRONTERA $y_1' = -y_2'$ $x = 270$ $z = 180$

$$\frac{1}{EI_1}\left(\frac{R_A x^2}{2} + C_1\right) = \frac{1}{EI_2}\left(\frac{R_B z^2}{2} + C_3\right)$$

$$\frac{2916\times10^5\ Kg\times cm^2}{\cancel{2\times10^6 \frac{Kg}{cm^2}}\ I_1} + \frac{C_1}{\cancel{EI_1}} = \frac{1944\times10^5\ Kg\times cm}{\cancel{2\times10^6\frac{Kg}{cm^2}}\times I_2} + \frac{C_3}{\cancel{EI_2}}$$

$$2916\times10^5 + C_1 = \frac{I_1}{I_2}\left[1944\times10^5 + C_3\right]$$

$$291600000 + C_1 = -\left[258940800 + 1.332 C_3\right]$$

$$\boxed{550540800 + C_1 + 1.332 C_3 = 0}$$

2DA CONDICION FRONTERA $y_1 = y_2 \Rightarrow x = 270$ $z = 180$

$$\frac{1}{EI_1}\left[\frac{R_A x^3}{6} + C_1 x\right] = \frac{1}{EI_2}\left[\frac{R_B z^3}{6} + C_3 z\right]$$

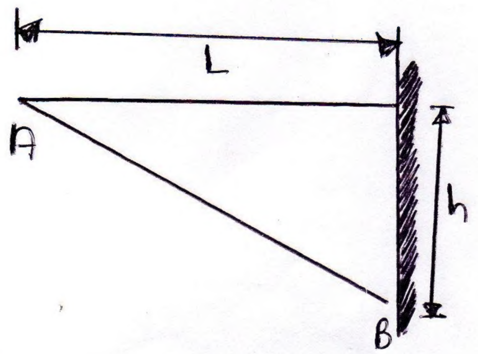
$$2.6244\times10^{10} + 270 C_1 = \frac{I_1}{I_2}\left[1.1664\times10^{10} + 180 C_3\right]$$

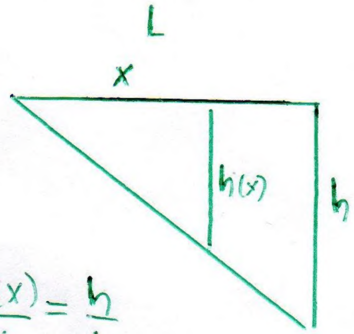
$$\boxed{1.0707552\times10^{10} + 270 C_1 - 239.76 C_3 = 0}$$

$$\boxed{\begin{array}{l} C_1 = -244010880 \\ C_3 = -230127567.6 \end{array}}$$

$y_1' = 1.429102703\times10^{-3}$ RPTA $y_1 = -1.190358$ cm RPTA

Determinar la deflexión en el punto "A" de la viga que se representa en la figura, debido a su peso propio. se sabe que el peso especifico del material de la viga es "γ", el módulo de elastiadad "E" y el anch de la viga es "b"





$$\frac{h(x)}{x} = \frac{h}{L}$$

$$h(x) = \frac{h}{L}x$$

$$V = \frac{1}{2} h(x)\, b\, x = \frac{hb}{2L} x^2$$

$$w = \frac{\gamma hb}{2L} x^2$$

$$I = \frac{b\,(h(x))^3}{12} = \frac{bh^3}{12L^3} x^3$$

$$IEy'' = -\frac{\gamma hb}{2L} x^2 \left(\frac{x}{3}\right) = -\frac{\gamma hb}{6L} x^3$$

$$Ey'' = \frac{-\frac{\gamma hb}{6L} x^3}{\frac{bh^3}{12L^3} x^3} = -\frac{2\gamma L^2}{h^2}$$

$$Ey' = -\frac{2\gamma L^2}{h^2} x + C_1$$

$$Ey = -\frac{2\gamma L^2}{h^2} \frac{x^2}{2} + C_1 x + C_2$$

Para $x = L$ $y' = 0$ $C_1 = \frac{2\gamma L^3}{h^2}$ ✓

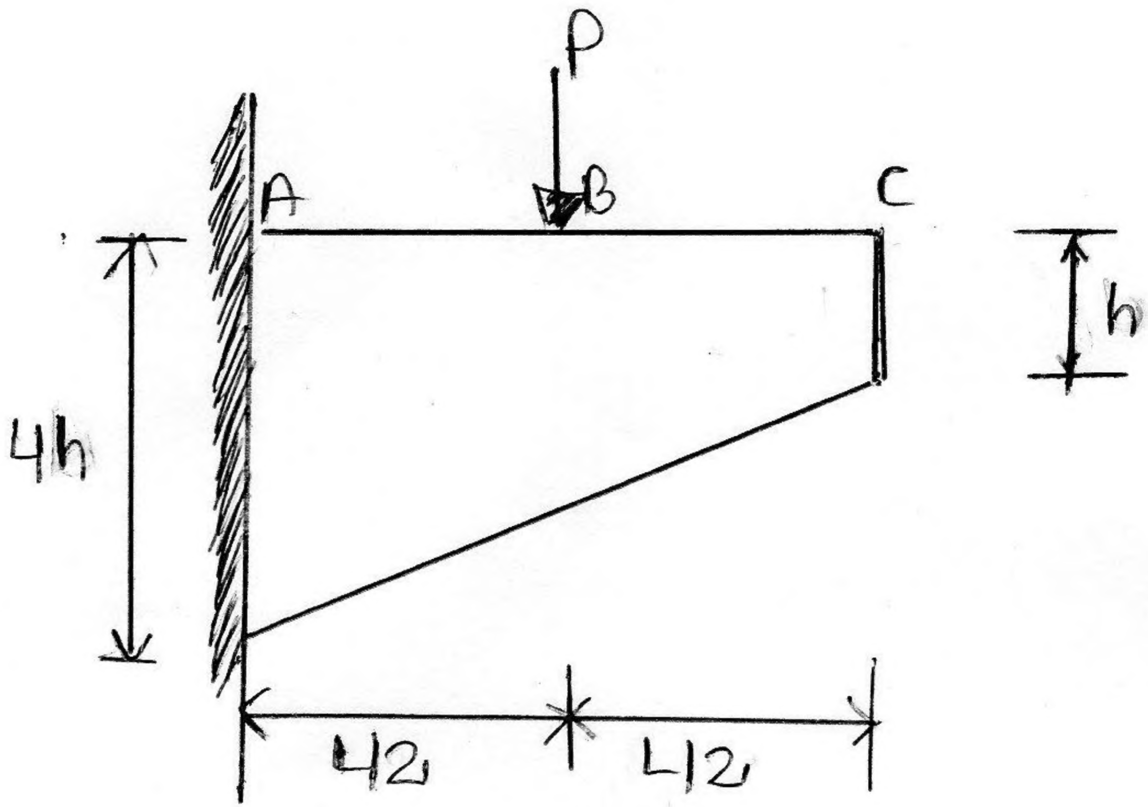
Para $x = L$ $y = 0$ $C_2 = -\frac{\gamma L^4}{h^2}$ ✓

deflexión "A" $\rightarrow x = 0$

$$Ey = -\frac{\gamma L^4}{h^2}$$

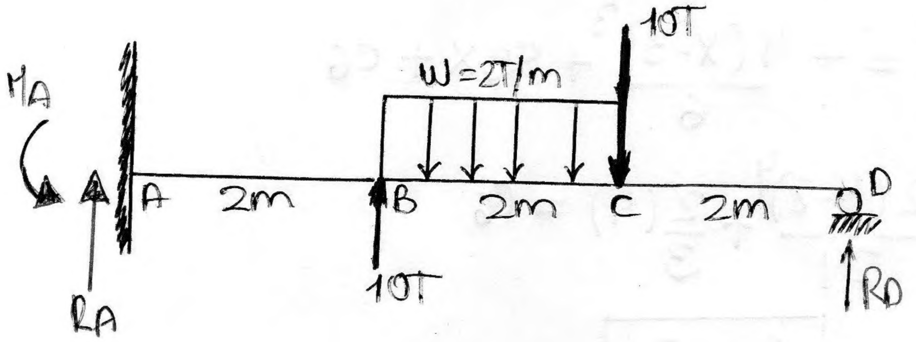
$$\boxed{y_A = -\frac{\gamma L^4}{E h^2}}$$ RPTA

La viga mostrada de sección variable es de un solo material con módulo de elasticidad E. Calcular la deflexión en el punto de aplicación de la carga P.



doble integración

Resolver la viga y determinar el valor de la deflexión máxima aplicando el método de la doble integración. EI = cte



Tramo $0 \leq x \leq 2$

$$I_1 E y_1'' = RAx - M$$

$$IEy_1' = \frac{RAx^2}{2} - Mx + \cancel{C_1}$$

$$IEy_1 = \frac{RAx^3}{6} - \frac{Mx^2}{2} + \cancel{C_1 x} + \cancel{C_2}$$

Para $x=0$ $y_1'=0$ $C_1=0$

Para $x=0$ $y_1=0$ $C_2=0$

Para $y'=0$ $x=$

Tramo $2 \leq x \leq 4$

$$IEy_2'' = RAx - M + 10(x-2) - w\frac{(x-2)^2}{2}$$

$$IEy_2' = \frac{RAx^2}{2} - Mx + 10\frac{(x-2)^2}{2} - w\frac{(x-2)^3}{6} + C_3$$

$$IEy_2 = \frac{RAx^3}{6} - \frac{Mx^2}{2} + 10\frac{(x-2)^3}{6} - w\frac{(x-2)^4}{24} + \cancel{C_3 x} + C_4$$

Para $x=2$ $y_1' = y_2' \rightarrow C_3 = 0$

Para $x=2$ $y_1 = y_2 \rightarrow C_4 = 0$

Para $y'=0$

tramo $4 \leq x \leq 6$

$$IEy_3'' = RAx - M + 10(x-2) - 4(x-3) - 10(x-4)$$

$$IEy_3' = \frac{RAx^2}{2} - Mx + 10\frac{(x-2)^2}{2} - 4\frac{(x-3)^2}{2} - 10\frac{(x-4)^2}{2} + C_5$$

$$IEy_3 = \frac{RAx^3}{6} - \frac{Mx^2}{2} + 10\frac{(x-2)^3}{6} - 4\frac{(x-3)^3}{6} - 10\frac{(x-4)^3}{6} + C_5 x + C_6$$

Para $x=4$ $y_2' = y_3'$

$$-w\frac{(4-2)^3}{6} = -4\frac{(x-3)^2}{2} + C_5$$

$$\frac{4}{2}(1)^2 - \frac{2(2)^3}{6} = C_5$$

$$\boxed{C_5 = -2}$$

Para $x=4 \quad y_1 = y_2$

$$-\frac{w(x-2)^4}{24} = -\frac{4(x-3)^3}{6} + C_5 x + C_6$$

$$\frac{4}{6}(4-3)^3 - \frac{2(4-2)^4}{24} + \frac{2}{3}(4) = C_6$$

$$\boxed{2 = C_6}$$

Para $x = 6m \quad y_2 = 0$

$$0 = \frac{RA(6)^3}{6} - \frac{M(6)^2}{2} + \frac{10}{6}(4)^3 - \frac{4(3)^3}{6} - \frac{10}{6}(2)^3 - \frac{2}{3}(6) +$$

$$\boxed{0 = 36RA - M_A + \frac{220}{3}}$$

$M_D = 0$

$$6RA + 10(4) - 4(3) - 10(2) - MA = 0$$

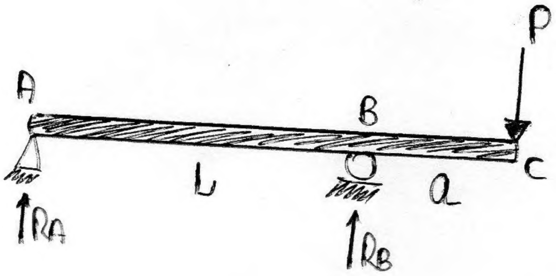
$$\boxed{6RA - MA + 8 = 0}$$

$RA = \frac{-53}{54}$ TN $\quad M = \frac{19}{9}$ TNxm $\quad RD = \frac{269}{54}$ TN

La viga parcialmente en voladizo de acero ABC soporta una carga concentrada P en el extremo C. Para la porción AB de la viga:

a) obtenga la ecuación de la curva elástica

b) determine la deflexión máxima



$R_B \cdot L = P(L+a)$

$R_B = \frac{P}{L}(L+a)\ (\uparrow)$

$R_A = \frac{Pa}{L}\ (\downarrow)$

$0 \leq x \leq L$

$IEy_1'' = R_A x$

$IEy_1' = R_A x^2/2 + C_1$

$IEy_1 = R_A x^3/6 + C_1 x + C_2$

$L \leq x \leq L+a$

$IEy_2'' = R_A x + R_B(x-L)$

$IEy_2' = R_A x^2/2 + R_B(x-L)^2/2 + C_3$

$IEy_2 = R_A x^3/6 + R_B(x-L)^3/6 + C_3 x + C_4$

Para $x=0$ $C_2=0$

Para $x=L$ $y_1=y_2=0$ $C_1=C_3$

$$0 = -\frac{Pa}{6L}(L^3) + C_1 L \Rightarrow \boxed{C_1 = \frac{PaL}{6}}$$

a) $IEy_1 = -\frac{Pa}{6}x^3 + \frac{PaL}{6}x$ ✓

b) deflexión máxima → giro = 0

$$0 = -\frac{Pa}{2L}x^2 + \frac{PaL}{6} \Rightarrow \frac{Pa}{2L}x^2 = \frac{PaL}{6} \Rightarrow x^2 = \frac{L^2}{\sqrt{3}^2}$$

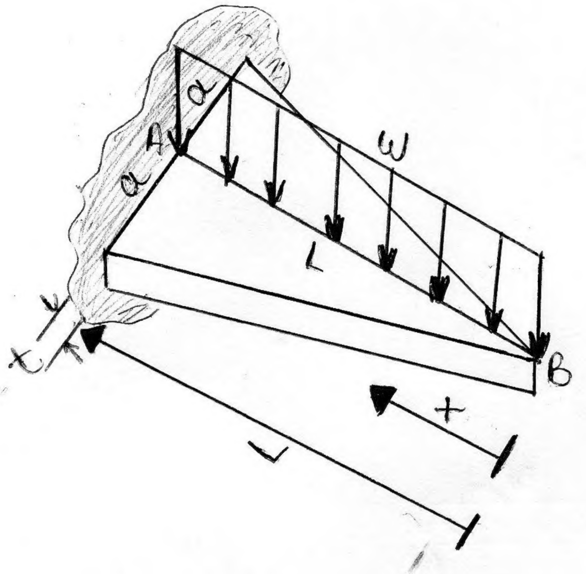
$$\boxed{x = 0.577L}$$

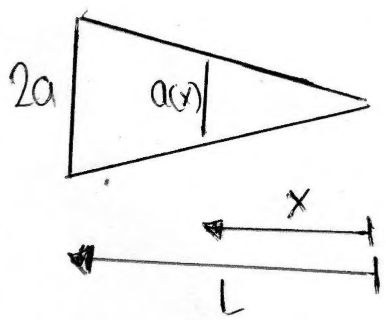
$$\therefore IEy_1 = -\frac{Pa}{6L}(0.577L)^3 + \frac{PaL}{6}(0.577L)$$

$$\boxed{y_1 = \frac{0.06414\,PaL^2}{IE}}$$ Rpta

doble integración

La viga mostrada tiene un espesor constante "t", esta en volodizo (empotramiento en "A") y es de un material de módulo "E". Determinar el valor de la deflexión máximo





$\frac{a(x)}{x} = \frac{2a}{L}$ $\Rightarrow$ $a(x) = \frac{2a}{L}x$

$I_x = \frac{bh^3}{12} = \frac{2at^3}{12L}x$

$I_x E y'' = -\frac{wx^2}{2}$

$Ey'' = -\frac{wx^2}{2I_x} = \frac{-\frac{wx^2}{2}}{\frac{2at^3 x}{12L}} = -\frac{12wLx^2}{4at^3x}$

√ $Ey'' = -\frac{3wxL}{at^3}$

√ $Ey' = -\frac{3wx^2}{2at^3}L + C_1$

√ $EY = -\frac{wx^3}{2at^3}L + C_1 x + C_2$

Para $x = L$ $\quad y' = 0 \quad y = 0$

$C_1 = \frac{3wL^3}{2at^3}$

$C_2 = \frac{wL^4}{2at^3} - \frac{3wL^4}{2at^3}$

$C_2 = \frac{-wL^4}{at^3}$

deflexion máxima

$\delta_{max} = \delta_B$

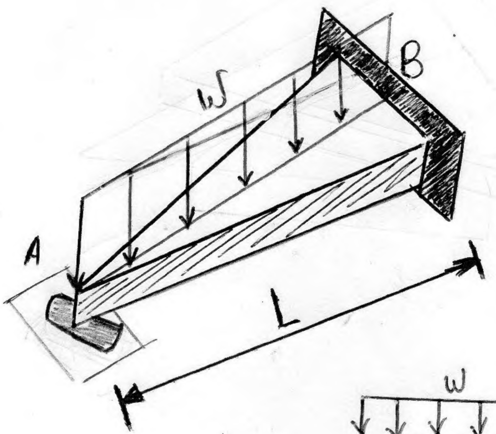
Para $x = 0$

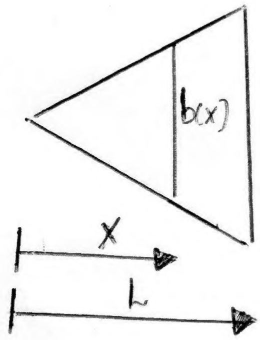
$EY = C_2$

$EY = \frac{-wL^4}{at^3} \qquad y = \frac{-wL^4}{Eat^3}$

$$\boxed{\delta_B = \delta_{max} = \frac{wL^4}{Eat^3} \downarrow}$$

La viga mostrada tiene un soporte sencillo en A y un empotramiento en B. El momento de Inercias de las secciones transversales varía linealmente desde cero en A a I_0 en B. Calcular la reacción en el soporte A debido a la carga lineal uniforme de intensidad w.

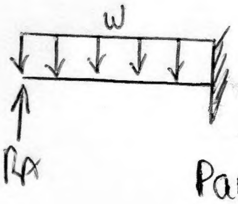




$$\frac{b(x)}{x} = \frac{b}{L}$$

$$b(x) = \frac{b}{L}x$$

$$I_x = \frac{b\,h^3}{12L}x$$



$$I_x E y'' = R_A x - \frac{w x^2}{2}$$

$$E y'' = \frac{R_A x}{\frac{bh^3}{12L}x} - \frac{\frac{w x^2}{2}}{\frac{bh^3 x}{12L}}$$

$$E y'' = \frac{12 L R_A}{bh^3} - \frac{12 L w}{2bh^3}x$$

$$E y' = \frac{12 L R_A}{bh^3}x - \frac{6wL}{bh^3}\frac{x^2}{2} + c_1$$

$$E y = \frac{12 L R_A}{bh^3}\frac{x^2}{2} - \frac{6wL}{bh^3}\frac{x^3}{6} + c_1 x + \cancel{c_2}^{\,0}$$

Para $x = 0$ $c_2 = 0$ $y = 0$

Para $x = L$ $y' = 0$

$$\frac{6wL}{bh^3}\left(\frac{L^2}{2}\right) - \frac{12 L R_A (L)}{bh^3} = c_1$$

$$\boxed{\frac{3wL^3}{bh^3} - \frac{12 R_A L^2}{bh^3} = c_1}$$

Para $x = L$ $y = 0$

$$0 = \frac{12 R_A L}{bh^3}\left(\frac{L^2}{2}\right) - \frac{6wL}{bh^3}\left(\frac{L^3}{6}\right) + \frac{3wL^3}{bh^3}(L) - \frac{12 R_A L^3}{bh^3}$$

$$0 = \frac{12 R_A L^3}{2} - \frac{wL^4}{1} + 3wL^4 - 12 R_A L^3$$

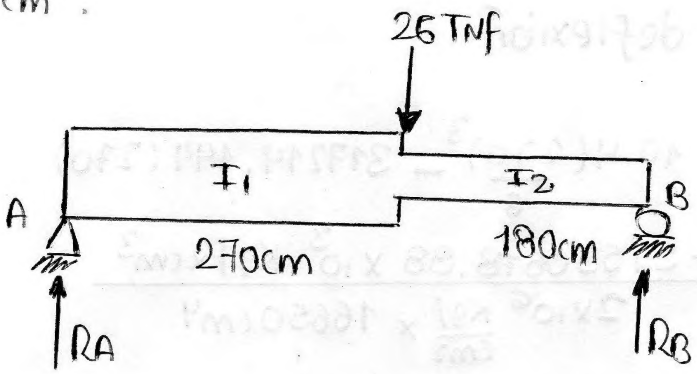
$$0 = -6 R_A L^3 + 2wL^4$$

$$6 R_A L^3 = 2wL^4$$

$$R_A = \frac{wL}{3}$$ RPTA.

Doble integración

Dada la viga simplemente apoyada de momento de inercia variable, determine el giro y flecha en el punto de la carga aplicada por el método de la doble integración. Se conoce $I_1 = 16650\ cm^4$ $I_2 = 12500\ cm^4$ $E = 2\times10^6\ Kf/cm^2$.



$$450 R_B = 26(270)$$
$$R_B = 15.6$$
$$R_A = 10.4$$

$$I_1 E y_1'' = 10.4x \qquad I_2 E y_2'' = 15.6 z$$
$$I_1 E y_1' = 10.4\, x^2/2 + C_1 \qquad I_2 E y_2' = 15.6\, z^2/2 + C_3$$
$$I_1 E y_1 = 10.4\, x^3/6 + C_1 x + \cancel{C_2}^{\,0} \qquad I_2 E y_2 = 15.6\, z^3/6 + C_3 z + \cancel{C_4}^{\,0}$$

Para $x = 270\ cm$ $\quad z = 180\ cm$ $\quad y_1 = y_2$

$$10.4\frac{x^3}{6} + C_1 x = \frac{I_1}{I_2}\left(15.6\frac{z^3}{6} + C_3 z\right)$$

$$10.4\left(\frac{270^3}{6}\right) + 270 C_1 = \frac{16650}{12500}\left(15.6\frac{(180)^3}{6} + 180 C_3\right)$$

$$\boxed{270 C_1 - 239.76 C_3 + 13919817.6 = 0}$$

Para $x = 270\ cm$ $\quad z = 180\ cm$ $\quad y_1' = -y_2'$

$$10.4\frac{x^2}{2} + C_1 = \frac{I_1}{I_2}\left(-15.6\frac{z^2}{2} + C_3\right)$$

$$10.4\left(\frac{180^2}{2}\right) + C_1 = \frac{16650}{12500}\left(-15.6\frac{(180)^2}{2} - C_3\right)$$

$$\boxed{C_1 + 1.332 C_3 + 715703.04 = 0}$$

hallando deflexión

$$EI_1 y_1 = 10.4\frac{(270)^3}{6} - 317214.144(270)$$

$$y_1 = \frac{-51530618.88 \times 10^3 \ Kgf \times cm^3}{2\times10^6 \ \frac{Kgf}{cm^2} \times 16650 \, cm^4}$$

$$\boxed{y_1 = -1.547466 \ cm} \quad RPTA$$

hallando giro

$$EI_1 y_1' = 10.4\frac{(270)^2}{2} - 317214.144$$

$$y_1' = \frac{61865.856 \times 10^3}{2\times10^6 \times 16650}$$

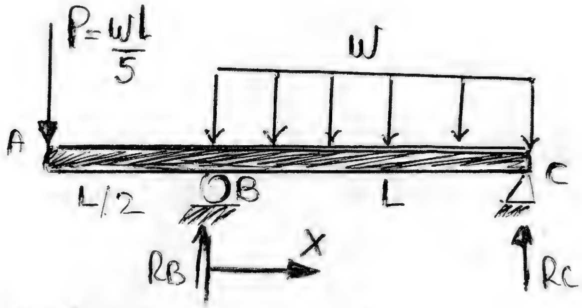
$$\boxed{y_1' = 1.857834 \times 10^{-3} \ rad} \quad RPTA$$

Para la viga y carga que se muestra en la figura encuentre :

a) Ecuacion de la curva elástica para el tramo BC de la viga

b) deflexion en la mitad de la luz

c) la pendiente en B



$$\frac{wL}{5}\left(\frac{3L}{2}\right)+\frac{wL^2}{2}=R_B(L)$$

$$R_B=\frac{4}{5}wL$$

$$R_A=\frac{2}{5}wL$$

$$IEy''=-\frac{wL}{5}\left(\frac{L}{2}+x\right)+\frac{4}{5}wLx-\frac{wx^2}{2}$$

$$IEy''=\frac{4}{5}wLx-\frac{wL^2}{10}-\frac{wLx}{5}-\frac{wx^2}{2}$$

$$IEy'=\frac{4wL}{10}x^2-\frac{wL^2}{10}x-\frac{wL}{10}x^2-\frac{wx^3}{6}+C_1$$

$$IEy=\frac{4wL}{30}x^3-\frac{wL^2}{20}x^2-\frac{wL}{30}x^3-\frac{wx^4}{24}+C_1x+C_2$$

$$IEy=\frac{wL}{10}x^3-\frac{wL^2}{20}x^2-\frac{w}{24}x^4+C_1x+C_2$$

Para $x=0$ $\quad C_2=0$

Para $x=0$ $\quad y=0$

$$0=\frac{wL}{10}(L^3)-\frac{wL^2}{20}(L^2)-\frac{w}{24}(L^4)+C_1(L) \rightarrow \boxed{C_1=\frac{-wL^3}{120}}$$ RPTA

a) $$IEy=\frac{wL}{10}x^3-\frac{w}{24}x^4-\frac{wL^2}{20}x^2-\frac{wL^3}{120}x$$ ✓

b) $$IEy=\frac{wL}{10}\left(\frac{L}{2}\right)^3-\frac{w}{24}\left(\frac{L}{2}\right)^4-\frac{wL^2}{20}\left(\frac{L}{2}\right)^2-\frac{wL^3}{120}\left(\frac{L}{2}\right) \rightarrow \boxed{y=\frac{-19wL^4}{1920EI}}$$

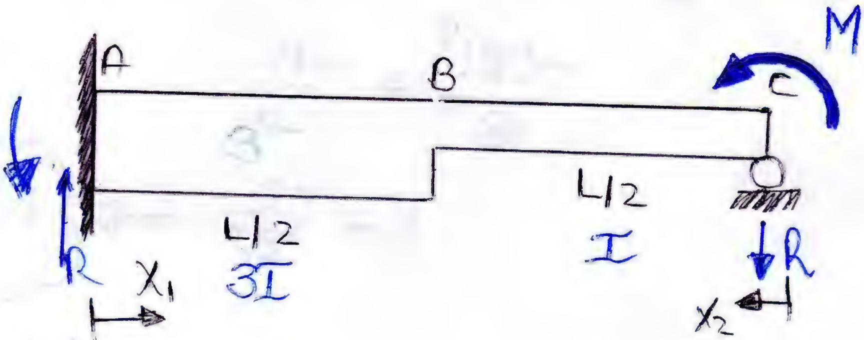
RPTA

c) giro en B $\quad x=0$

$$IE\theta_B=C_1 \Rightarrow \boxed{\theta_B=\frac{-wL^3}{120EI}}$$ RPTA

Para la viga ABC mostrada en la figura 1, se conocen E, I, M, L.
Se pide determinar, utilizando el método de doble integración lo siguie

a) El momento en el empotramiento A
b) La rigidez absoluta de la viga ABC.



$RL - M = M_A$

$$3EIy_1'' = RX - M_A$$
$$3EIy_1' = \frac{RX^2}{2} - M_AX + C_1$$
$$3EIy_1 = \frac{RX^3}{6} - \frac{M_AX^2}{2} + \cancel{C_1X} + \cancel{C_2}$$

Para $X=0$ $y_1=0$ $y_1'=0$
$C_2=0$ $C_1=0$

$$EIy_2'' = -RX + M$$
$$EIy_2' = -\frac{RX^2}{2} + MX + C_3$$
$$EIy_2 = -\frac{RX^3}{6} + \frac{MX^2}{2} + C_3X + \cancel{C_4}$$

Para $X_2=0$ $y_2=0$

Para $X_1=L/2$ $X_2=L/2$ $y_1=y_2$

$$\frac{Rx^3}{18} - \frac{M_Ax^2}{6} = -\frac{Rx^3}{6} + \frac{Mx^2}{2} + C_3x$$

$$\frac{Rx^3}{18} - \frac{RLx^2}{6} + \frac{Mx^2}{6} + \frac{Rx^3}{6} - \frac{Mx^2}{2} = C_3x$$

$$\frac{RL^3}{144} - \frac{RL^3}{24} + \frac{RL^3}{48} + \frac{ML^2}{24} - \frac{ML^2}{8} = \frac{C_3L}{2}$$

$$-\frac{RL^3}{72} \quad \frac{ML^2}{12} = \frac{C_3L}{2}$$

$$\boxed{-\frac{RL^2}{36} - \frac{ML}{6} = C_3}$$

Para $\quad x_1 = \frac{L}{2} \quad x_2 = \frac{L}{2} \quad y_1 = -y_2$

$$\frac{Rx^2}{6EI} - \frac{MAx}{3EI} = -\frac{Mx}{EI} + \frac{Rx^2}{2EI} - \frac{C_3}{EI}$$

$$\frac{Rx^2}{6} - \frac{RLx}{3} + \frac{Mx}{3} = -Mx + \frac{Rx^2}{2} + \frac{RL^2}{36} + \frac{ML}{6}$$

$$\frac{RL^2}{24} - \frac{RL^2}{6} - \frac{RL^2}{8} - \frac{RL^2}{36} = -\frac{ML}{2} + \frac{ML}{6} - \frac{ML}{6}$$

$$-\frac{5RL^2}{18} = -\frac{ML}{2}$$

$$M = \frac{5RL}{9} \longrightarrow RL = \frac{9M}{5}$$

$\therefore \quad RL - M = MA$

$$\frac{9M}{5} - M = MA \longrightarrow \boxed{MA = \frac{4M}{5}} \text{ Rpta.}$$

doble integración

Por el método de doble integración, de la viga mostrada, determinar:

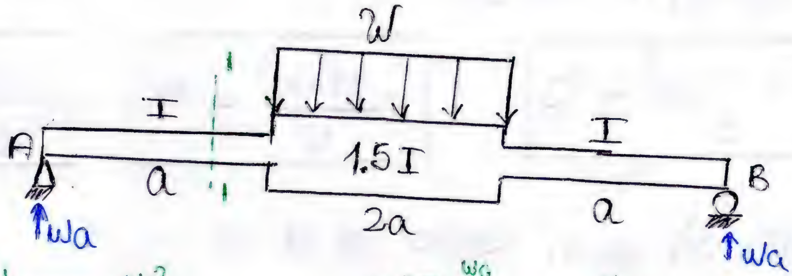
a) deflexión en el centro de luz de la viga

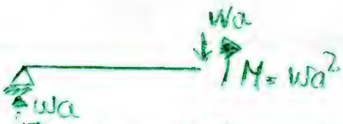
b) Pendiente en A

c) Pendiente en B

d) DFC

e) DMF



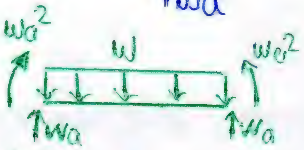


Tramo 1-1

$IEy_1'' = wax$

$IEy_1' = \frac{wax^2}{2} + C_1$

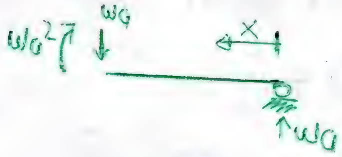
$IEy_1 = \frac{wax^3}{6} + C_1x + \cancel{C_2}$ 0



$1.5IEy_2'' = wax + wa^2 - \frac{wx^2}{2}$

$1.5IEy_2' = \frac{wax^2}{2} + wa^2x - \frac{wx^3}{6} + C_3$

$1.5IEy_2 = \frac{wax^3}{6} + \frac{wa^2x^2}{2} - \frac{wx^4}{24} + C_3x + C_4$



$IEy_3'' = wax$

$IEy_3' = \frac{wax^2}{2} + C_5$

$IEy_3 = \frac{wax^3}{6} + C_5x + \cancel{C_6}$ 0

• Para $x_1 = a \quad x_2 = 0: \quad y_1 = y_2 \ \wedge \ y_1' = y_2'$

$$\frac{wa}{6}(a^3) + C_1(a) = \frac{2}{3}C_4 \quad \wedge \quad \frac{wa}{2}(a)^2 + C_1 = \frac{2}{3}C_3$$

$$\frac{wa^4}{6} + C_1 a = \frac{2}{3}C_4 \quad \wedge \quad \frac{wa^4}{2} + C_1 a = \frac{2}{3}C_4 . a$$

$$\frac{wa}{3} = \frac{2}{3}C_3 a - \frac{2}{3}C_4 \Rightarrow \boxed{6wa = 12C_3a - 12C_4}$$

• Para $x_2 = 2a \quad x_3 = a \quad y_2 = y_3$

$$\frac{wa}{9}(2a)^3 + \frac{wa^2}{3}(2a)^2 - \frac{w}{36}(2a)^4 + \frac{2}{3}C_3(2a) + \frac{2}{3}C_4 = \frac{wa}{6}(a)^3 + C_5(a)$$

$$\boxed{\frac{29}{18}wa^4 + \frac{4}{3}C_3 a + \frac{2}{3}C_4 = C_5 . a}$$

• Para $x_2 = 2a \quad x_3 = a \quad y_2' = -y_3'$

$$\frac{wa}{3}(2a)^2 + \frac{2}{3}wa^2(2a) - \frac{w}{9}(2a)^3 + \frac{2}{3}C_3 = -\frac{wa}{2}(a)^2 + C_5$$

$$\boxed{\frac{41}{18}wa^4 + \frac{2}{3}C_3a = -C_5a}$$

$$\frac{70}{18}wa + 2C_3a + \frac{2}{3}C_4 = 0 \Rightarrow \boxed{-70wa = 36C_3a + 12C_4}$$

$$6wa^4 = 12C_3a - 12C_4$$
$$-70wa^4 = 36C_3a + 12C_4$$
$$-64wa^4 = 48C_3a$$

$\boxed{-\frac{4}{3}wa^3 = C_3}$ $\boxed{-\frac{11wa^4}{6} = C_4}$ $\boxed{-\frac{25}{18}wa^3 = C_5}$ $\boxed{-\frac{25wa^3}{18} = C}$

a) deflexión en el centro de la luz

√ Para $x_2 = a$

$$IEy_2 = \frac{wa}{9}(a)^3 + \frac{wa^2}{3}(a)^2 - \frac{w}{36}(a^4) - \frac{4}{3}wa^3(a) - \frac{11}{6}wa^4$$

$$\boxed{y_2 = -\frac{11wa^4}{4EI}}$$ Rpta

b) Pendiente en A

√ Para $x_1 = 0$

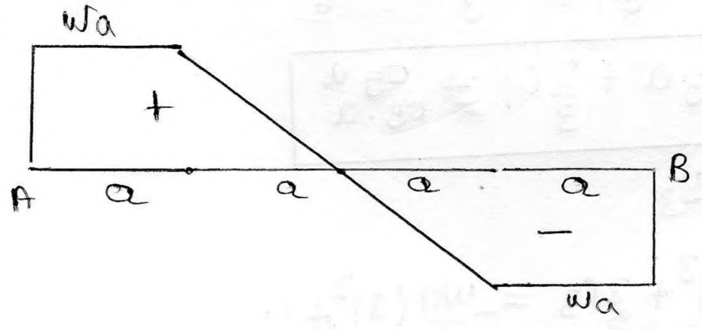
$IE\theta_A = C_1 \rightarrow$ $\boxed{\theta_A = -\frac{25wa^3}{18EI}}$ Rpta

c) Pendiente en B

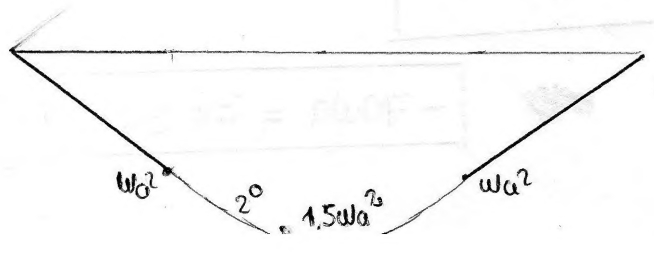
√ Para $x_3 = 0$

$IE\theta_B = C_5 \rightarrow$ $\boxed{\theta_B = \frac{-25wa^3}{18EI}}$ Rpta

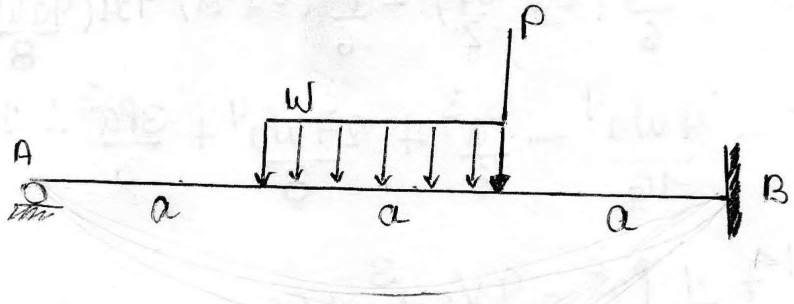
d) DFC



e) DMF



Resolver la siguiente viga y determinar el valor de la deflexión apl el método de la doble integración. considerar $EI = cte$.



Tramo $0 \leq x \leq a$

$EIy_1'' = V_A x$

$EIy_1' = \frac{V_A x^2}{2} + C_1$

$EIy_1 = \frac{V_A x^3}{6} + C_1 x + C_2$

$a \leq x \leq 2a$

$EIy_2'' = V_A x - \frac{w(x-a)^2}{2}$

$EIy_2' = \frac{V_A x^2}{2} - \frac{w(x-a)^3}{6} + C_3$

$EIy_2 = \frac{V_A x^3}{6} - \frac{w(x-a)^4}{24} + C_3 x + C_4$

Tramo $2a \leq x \leq 3a$

$EIy_3'' = V_A x - wa\left(x - \frac{3a}{2}\right) - P(x-2a)$

$EIy_3' = \frac{V_A x^2}{2} - \frac{wa}{2}\left(x - \frac{3a}{2}\right)^2 - \frac{P(x-2a)^2}{2} + C_5$

$EIy_3 = \frac{V_A x^3}{6} - \frac{wa}{6}\left(x - \frac{3a}{2}\right)^3 - \frac{P}{6}(x-2a)^3 + C_5 x + C_6$

✓ 1ERA CF $x=0$ $y=0$ ⇒ $C_2 = 0$

✓ 1ERA CC $x=a$ $y_1' = y_2'$ ⇒ $C_1 = C_3$

✓ 2DA CC $x=a$ $y_1 = y_2$ ⇒ $C_2 = C_4$

✓ 2DA CF $x=3a$ $y_3' = 0$ ⇒

$$0 = \frac{V_A(3a)^2}{2} - \frac{wa}{2}\left(3a - \frac{3a}{2}\right)^2 - \frac{P}{2}(3a-2a)^2 + C_5$$

$$0 = \frac{9a^2 V_A}{2} - \frac{9a^3 w}{8} - \frac{Pa^2}{2} + C_5$$

$$C_5 = \frac{9a^3 w}{8} + \frac{Pa^2}{2} - \frac{9a^2 V_A}{2}$$

✓ 3^{RA} C.F. $\quad x=3a \quad y_3=0 \Rightarrow$

$$0 = \frac{V_A(3a)^3}{6} - \frac{wa}{6}\left(3a-\frac{3a}{2}\right)^3 - \frac{P}{6}(3a-2a)^3 + 3a\left(\frac{9a^3w}{8} + \frac{Pa^2}{2} - \frac{9aV_A}{2}\right) + C_6$$

$$0 = \frac{9V_A}{2}a^3 - \frac{9wa^4}{16} - \frac{Pa^3}{6} + \frac{27wa^4}{8} + \frac{3Pa^3}{2} - \frac{27V_A a^2}{2} + C_6$$

$$0 = \frac{45}{16}wa^4 + \frac{4}{3}Pa^3 - 9V_A a^3 + C_6$$

$$\boxed{C_6 = 9V_A a^3 - \frac{45}{16}wa^4 - \frac{4}{3}Pa^3}$$

✓ 3^{RA} C.C. $\quad x=2a \quad y_2' = y_3'$

$$\cancel{\frac{V_A(2a)^2}{2}}^{0} - \frac{w(2a-a)^3}{6} + C_3 = \cancel{\frac{V_A(2a)^2}{2}}^{0} - \frac{wa(2a-\frac{3a}{2})^2}{2} - \cancel{\frac{P}{2}(2a-2a)}^{0} + C_5$$

$$-\frac{wa^3}{6} + C_3 = -\frac{wa^3}{8} + \frac{9a^3w}{8} + \frac{Pa^2}{2} - \frac{9a^2V_A}{2}$$

$$\boxed{C_3 = \frac{7}{6}a^3w + \frac{Pa^2}{2} - \frac{9a^2}{2}V_A}$$

✓ 4^{TA} C.C $\quad x=2a \quad y_2 = y_3$

$$\cancel{\frac{V_A}{6}(2a)^3}^{0} - \frac{w}{24}(2a-a)^4 + C_3(2a) = \cancel{\frac{V_A}{6}(2a)^3}^{0} - \frac{wa}{6}\left(2a-\frac{3}{2}a\right)^3 - \cancel{\frac{P}{6}(2a-2a)^2}^{0} + C_5x + C_6$$

$$\frac{14a^4w}{6} + \cancel{Pa^3} - \cancel{9a^3V_A} - \frac{wa^4}{24} = \frac{18a^4w}{8} + \cancel{Pa^3} - \cancel{9a^3} + 9V_A a^3 - \frac{45}{16}wa^4 - \frac{4}{3}Pa^3$$

$$\frac{1}{48}a^4w + \frac{14}{6}a^4w - \frac{1}{24}a^4w - \frac{18}{8}a^4w + \frac{45}{16}wa^4 + \frac{4}{3}Pa^3 = 9V_A a^3$$

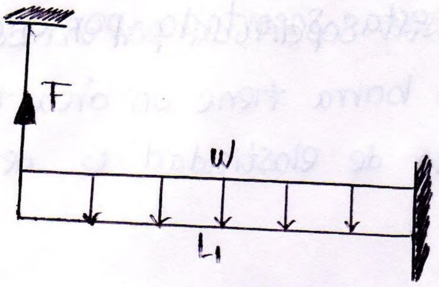
$$\frac{23}{8}a^4w + \frac{4}{3}Pa^3 = 9V_A a^3$$

$$\boxed{V_A = \frac{23}{72}aw + \frac{4}{27}P}$$

$$\boxed{V_B = \frac{49}{72}wa + \frac{23}{27}P}$$

$$\boxed{M_A = \frac{13}{24}wa^2 + \frac{5}{9}Pa}$$

Doble Integración



$$EIy'' = Fx - \frac{wx^2}{2}$$

$$EIy' = \frac{Fx^2}{2} - \frac{wx^3}{6} + C_1$$

$$EIy = \frac{Fx^3}{6} - \frac{wx^4}{24} + C_1x + C_2$$

Para $x = L_1$ $y' = 0$

$$0 = \frac{F}{2}(L_1)^2 - \frac{w}{6}(L_1^3) + C_1$$

$$\boxed{C_1 = \frac{wL_1^3}{6} - \frac{FL_1^2}{2}}$$

Para $x = L_1$ $y = 0$

$$0 = \frac{FL_1^3}{6} - \frac{wL_1^4}{24} + \frac{wL_1^4}{6} - \frac{FL_1^3}{2} + C_1$$

$$\boxed{C_2 = \frac{FL_1^3}{3} - \frac{wL_1^4}{8}}$$

Para $x = 0$

$$EIy = C_2 = \frac{FL_1^3}{3} - \frac{wL_1^4}{8} \Rightarrow y = -\left(\frac{wL_1^4}{8EI} - \frac{FL_1^3}{3EI}\right)$$

$$y = \frac{wL_1^4}{8EI} - \frac{FL_1^3}{3EI} \ (\downarrow)$$

$$\delta = \frac{FL}{A_2E_2} = \frac{wL_1^4}{8EI} - \frac{FL_1^3}{3EI}$$

$$\boxed{F = \frac{3wE_2A_2L_1^4}{8(3E_1I_1L_2 + E_2A_2L_1^3)}}$$ RPTA